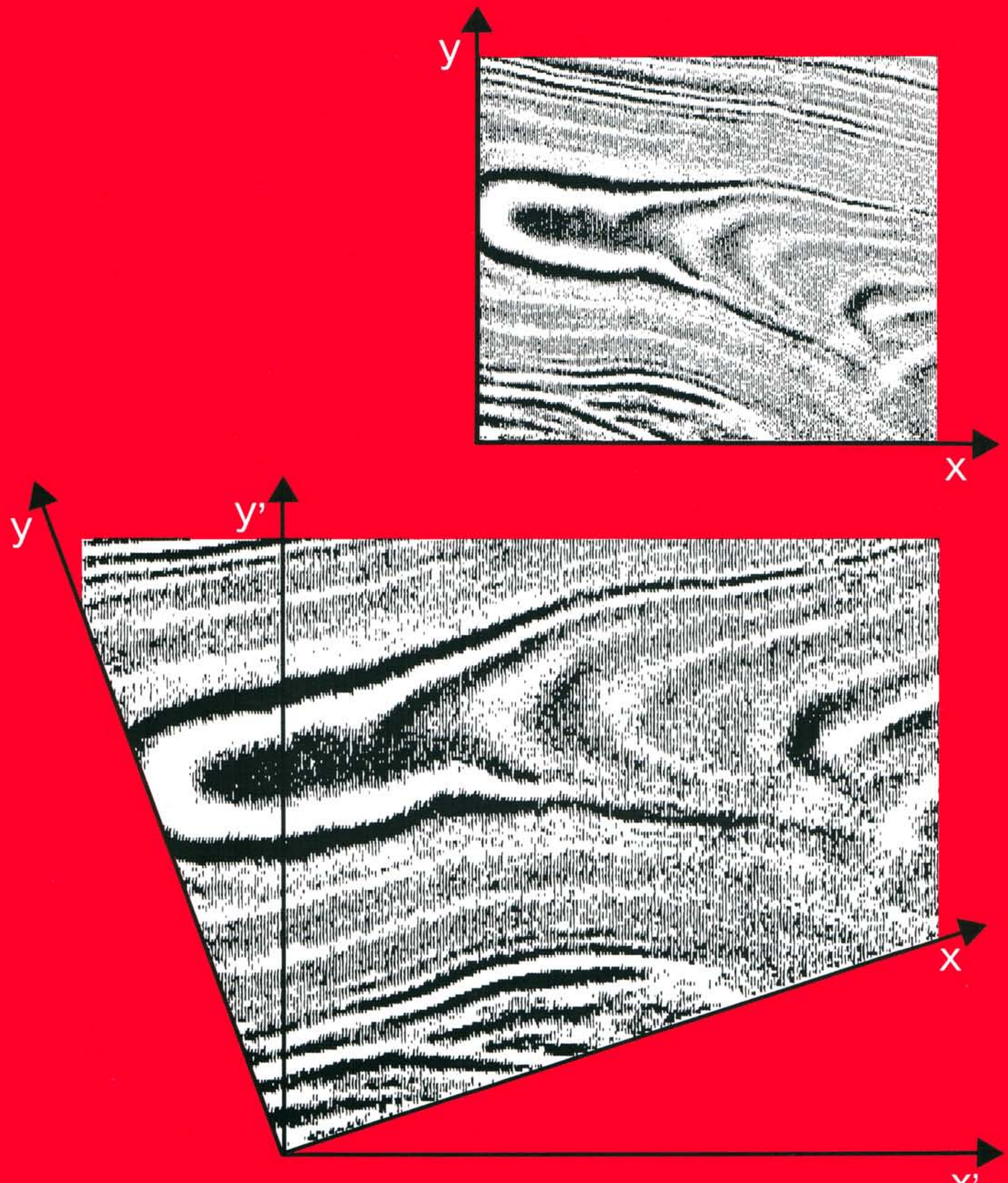
# PRAKLA-SEISMOS INFORMATION No. 68

## 1D Optimized Interpolation Operators for Single Trace Processing and 2D, 3D Interpolation of Seismic Data



## Introduction

In the interpolation of seismic data two different cases have to be considered, namely when

- the data are spatially aliased,
- the data are not spatially aliased.

The first case is described in PRAKLA-SEISMOS Information No 61 "Interpolation and Coherency Filtering, Two Applications of Seismic Pattern Recognition". The brochure you are now reading is concerned solely with the latter case, which involves preserving especially high frequency seismic information.

# A

## Optimum 1 D interpolation operators

The most important basic process with respect to these operators is 1 D interpolation of a given regularly sampled dataset to form another one with a smaller, larger or equal sampling interval. For the last mentioned only a time shift has to be applied to the dataset, whereby the time shift may be just a fraction of the sampling interval.

To meet the requirements of bandwidth preservation the operator design routine should include:

- An operator transition frequency band of variable width; for every width specified by the user a set of near-optimum operators should be generated.
- A number of different (fractional) shifts of the sampling interval. Thus, the design routine must supply a series of operators, which should generally be calculated in advance, stored in memory and used on request (table-driven interpolation).
- Operators with very uniform error behaviour or small overall error together with considerable bandwidth to avoid modulation effects.
- An additional anti-aliasing filter in the interpolation operators in order to improve the efficiency of the process. This filter is sometimes necessary when regridding involves increasing sampling interval.

There are three possibilities for dealing with the interpolation problem:

### a) Polynomial interpolation

This type of interpolation is usually implemented by convolution with simple operators derived from a cubic, quadratic or even linear interpolation formula. These operators are extremely short, nevertheless their amplitude and phase spectra render them quite unsuitable for all applications where broadband operation is critical. In the case of static corrections, in which the delay from interpolation usually varies randomly from trace to trace, it may be particularly disturbing to realize that the spectral response of these operators is highly dependent on the delay.

### b) Truncated sinc-interpolation

From the theoretical point of view the infinitely long sinc-operator is an appropriate operator for solving the interpolation problem. A finite impulse response of this operator is obtained by truncating it. However, in general it is not a very useful operator for interpolating discrete data. This can be seen in Figs 1 and 2, which show that the resulting frequency response of the truncated operator has considerable overshoot and undershoot as well as a slowly decaying ripple.

### c) Smooth band-limited interpolation (GRDPOL)

This new technique relies on a smoothed approximation of the operator in an extended spectral domain with subsequent transformation to the temporal domain and truncation at the intended length. Two methods have been developed for different purposes:

First method: This applies a soft flank to taper off the amplitude response smoothly from one to zero at the end of the usable band. This method of tapering is well suited to fulfil the anti-alias filter requirement; the operators are not pure interpolators, but are of mixed type with a well developed broad stopband and passband in which their phase response is determined by the interpolator characteristic.

Second method: The real part of the spectrum of any interpolation operator has a continuous transition over the Nyquist limit while the imaginary part changes its sign. This method tapers off the imaginary part only with the flank of the modified spectrum having a smooth continuation over the Nyquist limit.

Fig 3 shows the amplitude response of smooth interpolators designed by this new technique for a nominal delay of 0.5 samples with 12 and 48 filter points. To allow a direct comparison the scale is the same as that in Fig 1, in which the response of the truncated sinc-operators is displayed. Fig 4 shows the effective delay for 13 and 49 point operators with a nominal shift of 0.25 samples (compare with Fig 2).

Fig 5 demonstrates the flexibility and precision of the program by displaying the amplitude responses of 0.5 sample delay operators designed by the routine for 12, 24, 48, 96 filter points on a much finer scale. Obviously, these operators do not exhibit equi-ripple behaviour; the best possible tradeoff between steepness of the transition flank and flatness of the passband roof can be reached by allowing an overshoot maximum of about 1 % followed by a rapid flattening of the roof. It can be seen that these specifications are closely followed by all operators: the roof is virtually flat prior to an overshoot maximum of well below 1.2 %.

Fig 6 shows the combined interpolation/highcut operators designed by the first method. All displayed operators have their 50 % amplitude point at half the Nyquist frequency and a nominal shift of 0.5 samples. The amplitude spectra of a 24 and a 96 point operator are shown.

Truncated Sinc-Interpolator

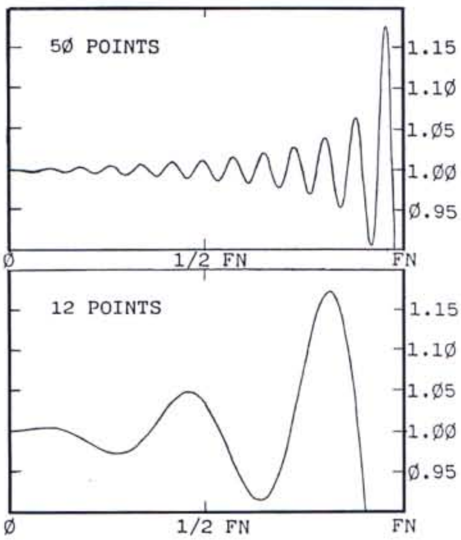



*Fig 1: Amplitude spectra of sinc-interpolators (0.5 samples nominal). Truncation: 12 and 50 points. Phase error is zero.*

Smooth Interpolator

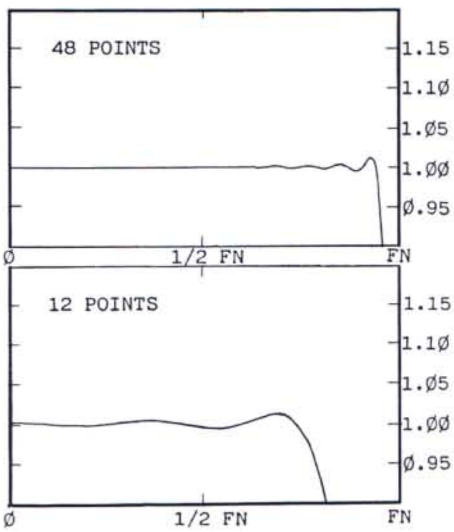



*Fig 3: Amplitude spectrum of smooth interpolators (0.5 samples nominal). Compare with Fig 1.*

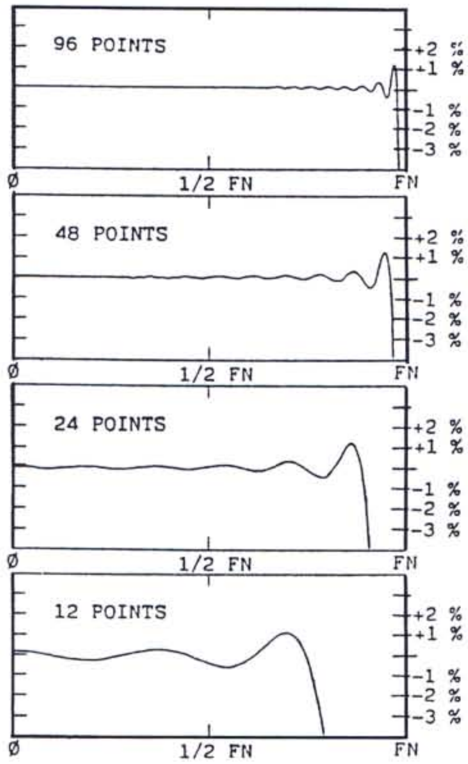



*Fig 5: Amplitude error spectra of smooth 0.5 sample interpolators.*

Truncated Sinc-Interpolator

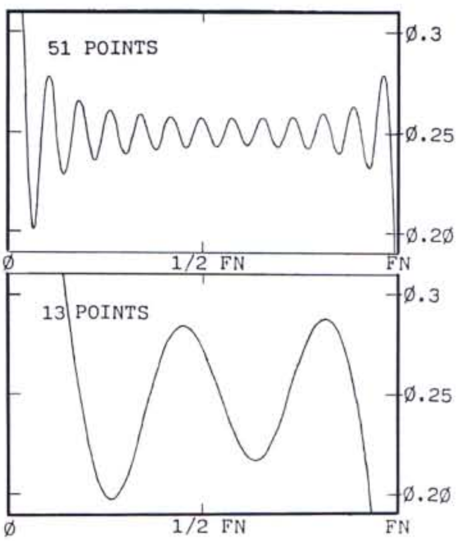



*Fig 2: Effective time shift vs frequency for sinc-interpolators (0.25 samples nominal).*

Smooth Interpolator

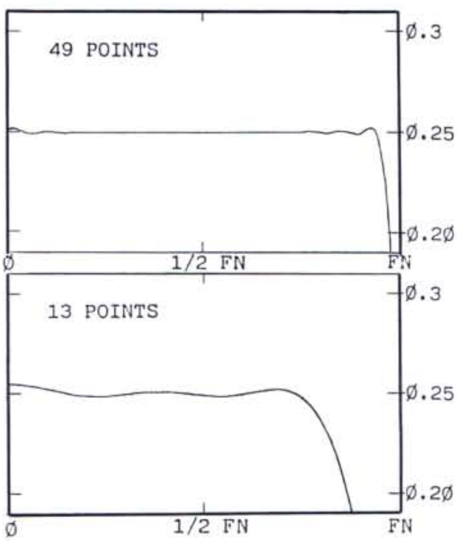



*Fig 4: Effective time shift vs frequency for smooth interpolators (0.25 samples nominal).*

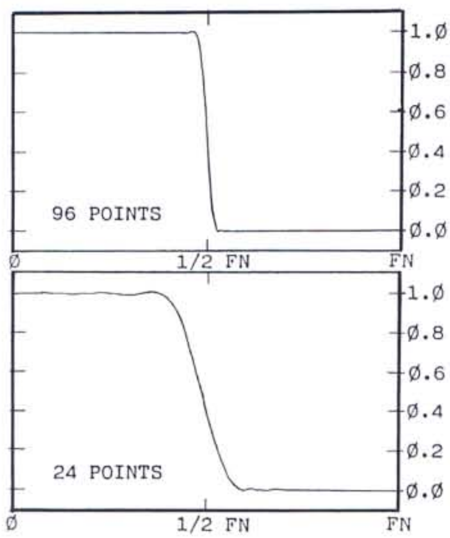



*Fig 6: Amplitude spectra of combined interpolation/highcut operators. Shift is 0.5 samples and nominal 50 % cutoff point is at 0.5 FN.*

# B

## Interpolation of a 2D dataset

2D interpolation with 1D optimized smooth interpolation can be simply performed in a horizontal direction of a 2D dataset. The data may be interpolated to a denser or less dense grid, including anti-alias filtering, or to a shifted grid with equal intervals. 2D interpolation is a special case of 3D interpolation, which is considered below in detail.

# C

## Interpolation of a 3D dataset (non-aliased)

In 3D seismic work there is often the problem of transforming a set of 3D data from one coordinate system to another. For example such a transformation is necessary to merge two overlapping 3D survey areas with different acquisition geometries so as to obtain a uniform post-stack geometry for subsequent 3D migration or computer-aided interpretation. Assuming that both survey areas lie on regular grids, the required transformation needed is composed of translation, dilation and rotation of the basic grid cell.

The problem is illustrated in Fig 7. Here.the original 3D grid has to be transformed into another grid, which has been rotated and which shows different spatial sampling intervals compared to those in the original coordinate system.

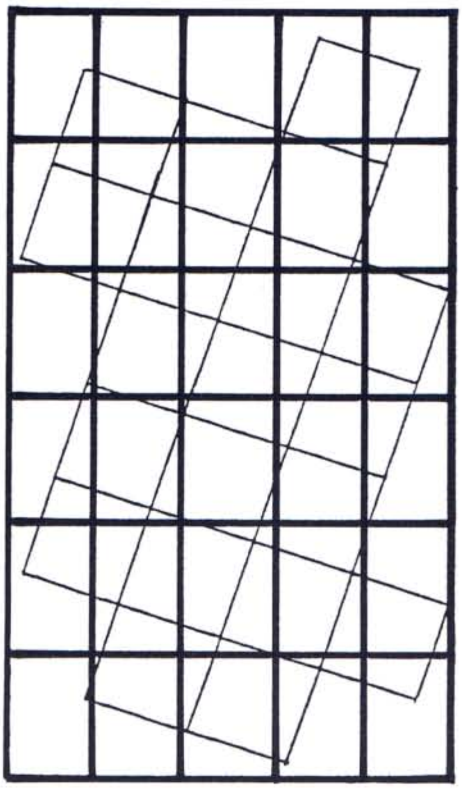


*Fig 7: Seismic data have to be transformed from the original (darker) coordinate system to the new rotated grid to enable data with different acquisition geometries to be merged.*

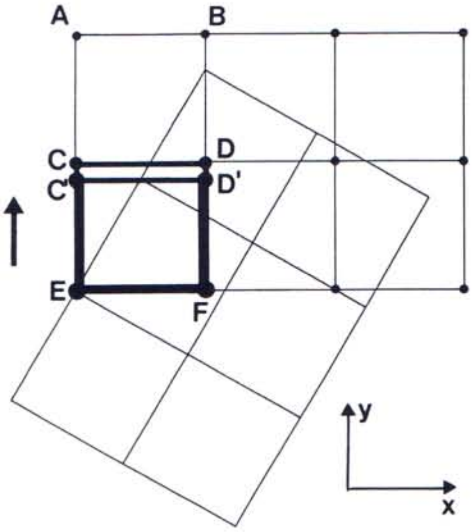



*Fig 8: Interpolation step 1: Obtain new y-distance as the projection of the output y-vector on the input y-vector (an inline dilation).*

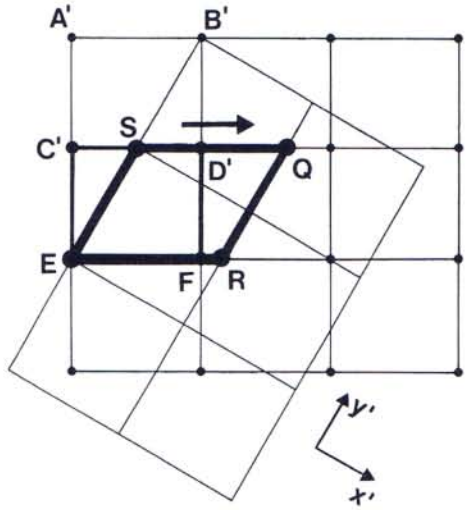



*Fig 9: Interpolation step 2: Move to the intersections of the input x-lines with the output y-lines (an inline translation/dilation).*

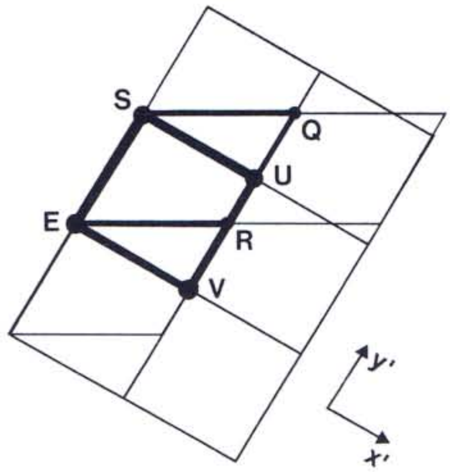



*Fig 10: Interpolation step 3: A final shearing (inline translation) generates the output grid. The procedure is exact if all data are well-sampled on input and output grid.*

## Approximation to the 2 D sinc-operator

Assuming the input and output grid fulfil the requirements of the 2 D sampling theorem, complete restoration of the data can be achieved in the new grid by a two-dimensional filter which represents a generalization of the well-known one-dimensional sinc-type interpolator. This 2 D operator could be applied to data of equal traveltime, in other words to time-slice data. As the points of the target grid usually appear to be quite irregular compared to the input grid, the 2 D filter coefficients have to be computed afresh for every output trace to be generated. Consequently, this 2 D interpolation is not an easy task for any existing computer system.

The 2 D operator can, however, be separated into a product of two one-dimensional operators; the cell translation and the dilation can be split up into regridding in the x-direction followed by a regridding in the y-direction, which is very similar to two-pass 3 D migration. It can be shown for cell rotation that this transformation is represented by a shearing in the x-direction followed by a 90-degree rotation and then by a shearing in the y-direction.

This type of interpolator is installed in the PRAKLA-SEISMOS GEOSYS system. Runtime observations have shown that a data volume of 400 000 traces can be processed on a vector computer system in mere hours (wall-clock time).

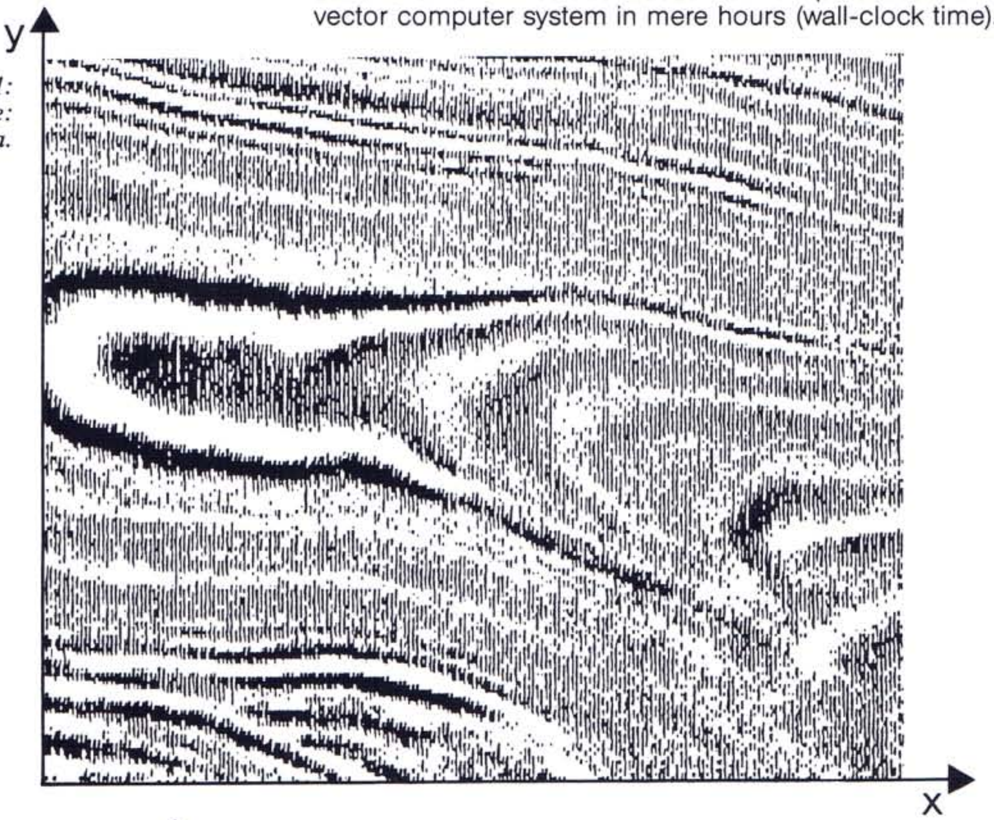



*Fig 11: Part of time slice: Original data.*

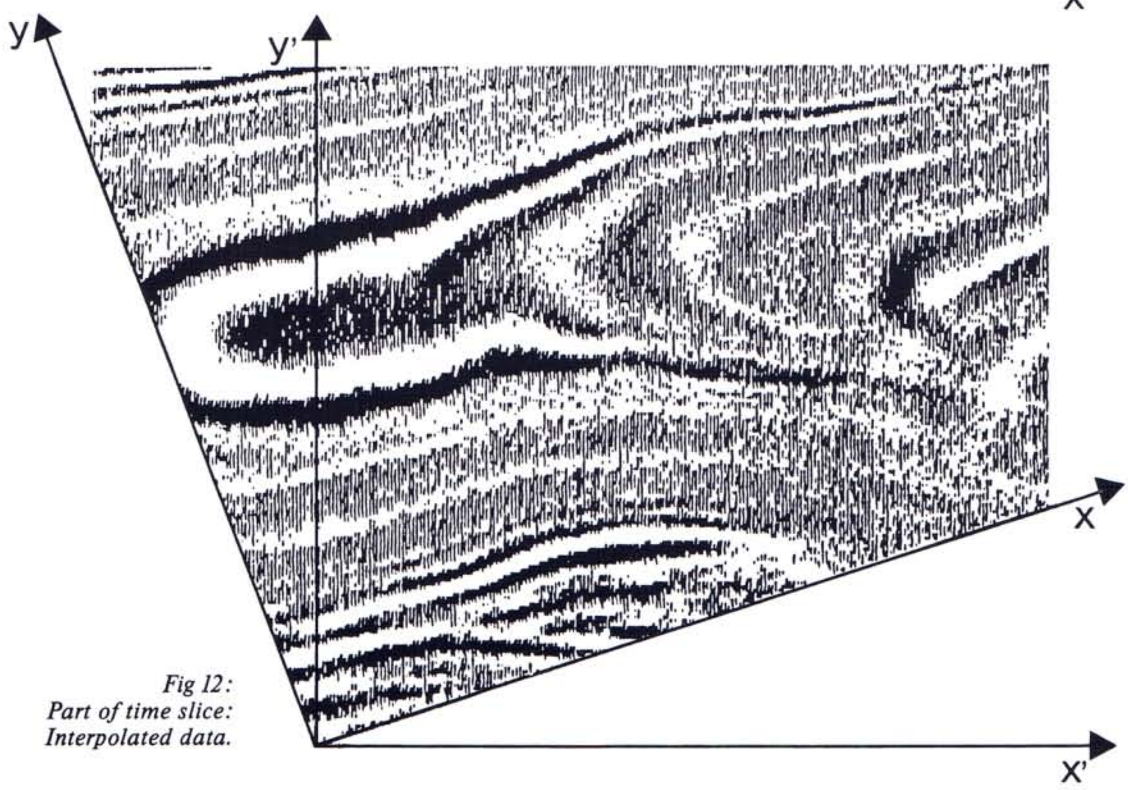



*Fig 12: Part of time slice: Interpolated data.*

The sequential application of 1 D operators is explained in Fig 8 to Fig 10. In Fig 8 the original grid in y is regridded in the y-direction (eg from A, B, C, D, to A', B', C', D'), then in a second step linear interpolation is performed in the x-direction (eg S, Q, R are interpolated). Finally in step three the grid points U and V are interpolated in the new y'-direction to construct the grid points of the new cell E, S, U, V.

In the following example the run comprised some 500 000 traces. Fig 11 shows part of a time slice of the input data; Fig 12 depicts the same part after an affined grid transformation. Both figures refer to the x–y orientation of the respective grids and therefore the second figure is rotated. The transformation comprised rotation of about 18 degrees, alteration of the coordinate origin and a change of the trace interval from 25 m to 20 m. Upon closer inspection it is recognized that all details are well restored (some small visual discrepancies are caused by the asymmetry of the time-slice display in seismic mode).

This result is confirmed by looking at the sections in Figs 13 and 14. Fig 13 was produced by picking a y-line of the target geometry and sorting all traces of the input data next to this line into one section. This diagonally sorted section can therefore be interpreted as the result of "next neighbour" or stepwise constant interpolation (without changing trace interval). As is to be expected, this section looks somewhat noisy and incoherent. Fig 14 shows the same section taken from the output of the grid transformation. It is clearly seen that the resolution of fine details is enhanced, while at the same time the signal to noise ratio and the coherency are significantly improved.

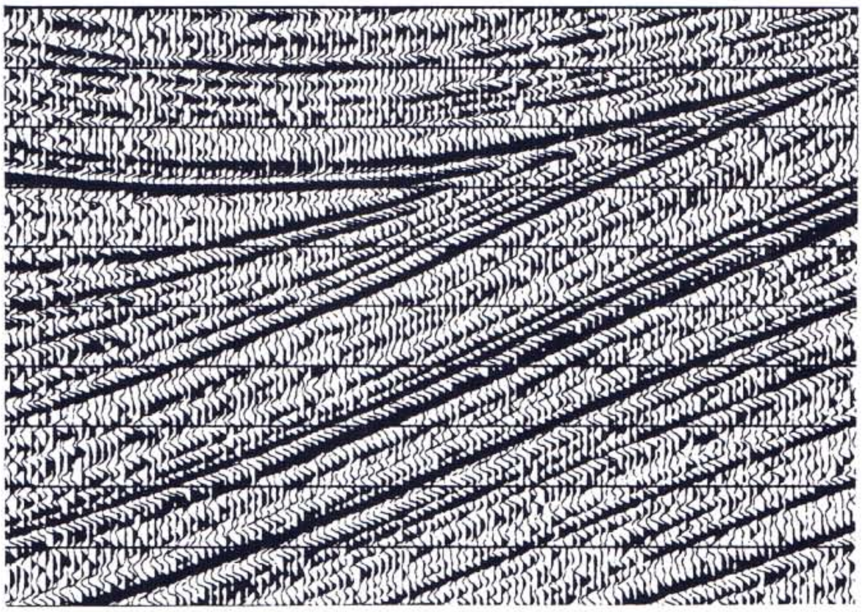


*Fig 13:*
*"Diagonal sorting" of original data following a proposed output y-line.*

*Fig 14:*
*y-line of interpolated data corresponding to the "diagonal sorting" of the input.*

PRAKLA-SEISMOS
**PRAKLA-SEISMOS AG · BUCHHOLZER STR. 100 · P.O. BOX 51 05 30**
**D-3000 HANNOVER 51 · FEDERAL REPUBLIC OF GERMANY**
**PHONE: (5 11) 64 20 · TELEX: 9 22 847 + 9 22 419 + 9 23 250 · TELEFAX: (5 11) 6 47 68 60**

6000 0890

ISSN 0933-7768